Scaling in the Diffusion Limited Aggregation Model: towards ultimate growth probability.

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Nowdays the field of statistical physics of equilibrium critical phenomena is clearly understood [1]. Critical exponents which characterize the behavior of a system near a critical point could be derived with the help of various well-established theories. These theories also explain why various systems behave very similarly near a critical point. From this standpoint, the universality concept is a rather well-grounded notion supported by numerous experiments and theoretical predictions.

In contrast, understanding nonequilibrium critical phenomena is still an open problem. In systems that belong to a self-organized criticality class [2] there is no control parameter that determine proximity to the critical point. Instead such systems exhibit fractal and scaling behavior in a very wide range of system properties. In 1981 Witten and Sander introduced a new model, called the diffusion limited aggregation (DLA) model [3], which describes pattern formation by a series of simple rules. These rules are essentially a Brownian random walk of single particle which wanders around an aggregate until it sticks to it after a collision. The repetition of these trivial rules leads to the formation of a very complex fractal object. Although the DLA model does not have any direct control parameter, like the percolation probability, it still has the main features of criticality: it exhibits fractal and scaling properties.

It is a long standing question wheather DLA model exibits universality. Numerous studies searched for evidence of universality by calculating the fractal dimension on different latices. Recent research [4] shows that there is no such universality in this model: the fractal dimension of on-lattice clusters approaches 3/2, while off-lattice clusters have a dimension close to 1.71.

Nevertheless, there are some indications of universality of different nature: several different models such as DLA, dielectric breakdown model, viscous fingering [5], and Laplacian growth have similar fractal dimension. But in contrast to ordinary systems like Ising and Potts models, where typicaly two critical exponents coincide, here seems to be only one critical exponent (namely fractal dimension) that corresponds to DLA model.

In this work we address the following important question. Since the convergence of the DLA model is very slow there is some uncertainty about the actual value of the fractal dimension of the DLA cluster. To address this question we suggest a simple but complete picture of DLA growth based on analyzing the probability density function $P(r, N)$ for the next particle to be attached at a distance r from the origin. We propose a scale-invariant form for the function $P(r, N)$

$$P(r, N) = \frac{1}{R_{\text{dep}}} f\left(\frac{r}{R_{\text{dep}}(N)}\right).$$

and check our assumptions numerically. It immediately follows [7] from our theory that there is only one scaling exponent D and there is no multiscaling in an asymptotically large DLA cluster.

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